Time frequency and time scale transforms, a comparison of different tools in the vibratory analysis of the induction motors

Cyrille caironi (1)(2)(3) ; Lionel Durantay (1) ; Valérie Kromer (3) ; Michel Gueury (3)
(1) Alstom moteurs.4, rue de la Rompure54250 Champigneulles, France, cyrille.caironi@powerconv.alstom.com
(2) GREEN, UHP Nancy 1, BP 239, 54506 Vandoeuvre les Nancy, abderrezak.rezzoug@green.uhp-nancy.fr
(3) ERIN, UHP, Esstin, 2 rue Jean Lamour 54500 Vandoeuvre

Abstract

In this paper, an application of the time frequency and time scale transforms to the vibratory analysis of the ball bearings in the induction motors are presented. The mathematical tools and their application on an experimental case are also presented. After, the utility of these tools in an industrial process are discussed.

1 Introduction

Time frequency and time scale transforms have been used for the vibratory analysis for many years. In Alstom moteurs, we tried to know how to use these type of tools for the maintenance of rotating electrical machines.

The first step is the understanding of the mathematical tools and of their limits. It the first part of this paper. The second part presents an experimental application on a rolling bearing of an induction motor and comments of the results.

2 Theory and implementation of the time-frequency transforms

The FFT enables to analyze the signal in the time domain or in the frequency domain. The transforms that we describe in this chapter enable to work in both domains. To do that, we have to work on the complex form of the signal. This form is

$$\tilde{s}(t) = s(t) + jH[s(t)] \qquad (1)$$

where H[s(t)] is the Hilbert transform of the real signal.

We present the Short Time Fourier Transform (STFT) and the Wigner Distribution (WD). They are described with test signals to evaluate them and to make a choice for the PD analysis. The signals chosen to do this test are sum of sinusoids and linear chirp :

$$x(t) = \sin(2pf_1 t) + \sin(2pf_2 t) \qquad (2)$$

where f1=200 Hz and f2=400 Hz

$$y(t) = \sin(2pft) \qquad (3)$$

where f ranges from 50 Hz to 350 Hz. The last test signal is the sum of x and y :

$$z(t) = x(t) + y(t) \qquad (4)$$

2.1 Short time Fourier transforms

To study the properties of a signal at time t, one emphasizes the signal at that time and suppresses the signal at other times. This is achieved by multiplying the signal by a window function [1], centered at t, to produce the modified signal.

$$s_t(t) = s(t).h(t - t) \qquad (5)$$

The modified signal is a function of two times, the fixed time we are interested in, t, and the running time, τ. The window function is chosen to leave the signal more or less unaltered around time t. Every point outside this window is put at zero to suppress the signal for time distant from the time of interest.

The term “window” comes from the idea that we are seeking to look at only a small piece of the signal as when we look out of a real window and see only a relatively small portion of the scenery. In this case, we want to see only a small portion.

Since the modified signal emphasizes the signal around time t, the Fourier transform will reflect the distribution of the frequency around that time,

$$S_t(w) = \frac{1}{\sqrt{2p}} \int e^{-jwt}.s_t(t).dt \qquad (6)$$

where ω is the pulsation of the signal, ω=2πf (f is the frequency in Hz). Following the formula (1),

$$S_t(w) = \frac{1}{\sqrt{2p}} \int e^{-jwt}.s(t).h(t-t).dt \qquad (7)$$

the energy density spectrum at time t is therefore

$$P_{SP}(t,w) = \left|S_t(w)\right|^2 = \left|\frac{1}{\sqrt{2p}} \int e^{-jwt}.s(t).h(t-t).dt\right|^2 \qquad (8)$$

For each different time we get a different spectrum and the totality of these spectra is the time-frequency distribution, P_{SP}. It goes under many names, depending on the field. In this paper, we use the most common phraseology, “spectrogram”. The figures 1b, 2b and 3b give examples of test signal defined as above.

Since we are interested in analyzing the signal around the time t, we have chosen a narrow window peaked around t. Hence, the modified signal is short and its Fourier transform, equation (7), is called Short-time Fourier transform. When we want to estimate time properties for a given frequency. We do not take short times but long ones. In this case, the windows function on the frequency is short to respect the uncertainty principle of Heisenberg. We can define a Short Frequency Fourier Transform by

$$s_w(t) = \frac{1}{\sqrt{2p}} \int e^{jwt}.S(w').H(w'-w).dw' \qquad (9)$$

If we relate the window function in the time h(t) and the windows function in the frequency H(ω) by

$$H(w) = \frac{1}{\sqrt{2p}} \int h(t).e^{-jwt}dt \qquad (10)$$

then

$$S_t(w) = e^{-jwt}.s_w(t) \qquad (11)$$

The short time Fourier transform is the same as the Short Frequency Fourier Transform except for the phase factor $e^{-j\omega t}$. Since the distribution is absolutely square, the phase factor does not enter into and either the short-time Fourier transform or the Short Frequency Fourier Transform can be used to define the distribution,

$$P(t,w) = \left|S_t(w)\right|^2 = \left|s_w(w)\right|^2 \qquad (12)$$

this shows that the spectrogram can be used to study the behavior of time properties at a particular frequency. This is done by choosing a narrow window in frequency or likewise by taking a broad window in time. Therefore we can make a narrowband or a wideband spectrogram in choosing size of the window in the time domain.

2.2 Wigner-Ville Disdributions

The Wigner distribution (WD) is the prototype of distributions that are qualitatively different from the spectrogram. Wigner’s original motivation for introducing it was to be able to calculate the quantum correction to the second virial coefficient of a gas. The Wigner distribution was introduced into signal analysis by Ville. The Wigner transform in terms of the signal is

$$W(t,w) = \frac{1}{2p} \int s^*(t-\frac{t}{2}).s(t-\frac{t}{2}).e^{-jwt}.dt \qquad (13)$$

The Wigner distribution is said to be bilinear in the signal because the signal enters twice in its calculation. It is to be noticed that to determine the properties of the Wigner distribution at a time t we mentally fold the left part of the signal over the right one to see if there is any overlap.

As we can see on the signal y(t) defined by the equation 3 (figure 2a) the Wigner distribution (figure 2c) has a better contrast than the Short Time Fourier Transform (figure 2b). It is interesting in the strongly non stationary signals. The figure 1 presents the sum of sinusoids x(t) defined by the equation 2. The STFT gives only both frequencies but with broad band. The WD gives the both frequencies with a narrow band but we can see a lot of interference terms especially in the 300 Hz band. These terms of interference are mathematical artifacts generated by the distribution.

The sum of both signals is defined as the equation (4) of the signal z(t). Its Wigner distribution is

$$W_z(t,\omega) = W_x(t,\omega) + W_y(t,\omega) + W_{xy}(t,\omega) + W_{yx}(t,\omega) \quad (14)$$

The cross Wigner distribution is complex. However, $W_{xy} = W_{yx}^*$ and, therefore, $W_{xy}(t,w) + W_{yx}(t,w)$ is real. Hence

$$W_z(t,w) = W_x(t,w) + W_y(t,w) + 2.Re\{W_{xy}(t,w)\} \quad (15)$$

We see that the WD of a sum of two signals is not the sum of the WD of each signal but has the additional term2.Re{Wxy(t,ω)}. It is our term of interference.

On one hand the signal z(t) (equation (4) figure 3a) is easily readable with the spectrogram (figure 3b). On the other hand the artifacts generated by the terms of interference blur the information on figure 3b. the work inside the classes of Cohen and the treatment of the ambiguity function enable to reduce this problem. But it is not the objective of this paper.

We can see that the easier tools in the time frequency is the Time Frequency space is the Short Time Fourier Transform. If the results of this transform are enough, it isn't necessary to use others. But, we have to remember the disadvantages of this transform. The first one is the non orthogonality of the transform and the impossibility of perfect reconstruction of the signal. The second one is the difficulty of the choice of the windows.

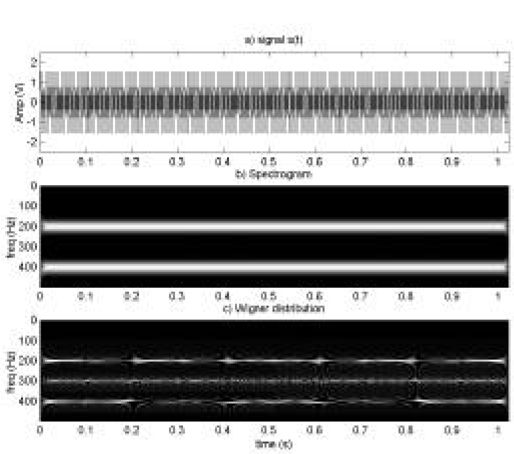

Figure 1: Spectrogram and WV distribution of the signal x(t)

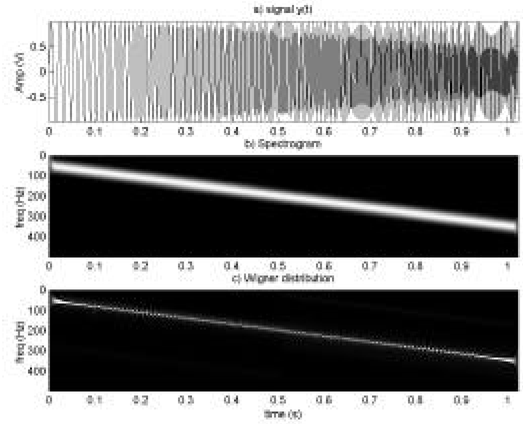

Figure 2: Spectrogram and WV distribution of the signal x(t)

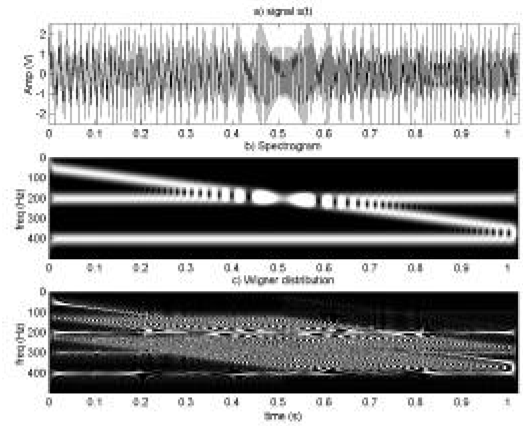

Figure 3: Spectrogram and WV distribution of the signal x(t)

Nevertheless, we decided to use Short Time Fourier Transform for the Partial Discharges Analysis. Indeed, we have the better results with this tool in term of facility and readability.

3 Theory and implementation of the time-scale transforms

The main difference between the Time-Frequency transforms and the time-scale transforms (or Wavelet Transforms) is the treatment of the analyzing function the signal. While the Time frequency compromise between time and frequency information can be useful, the drawback is that once we choose a particular size for the time window, that window is the same for all frequencies. Many signals like PD require a more flexible approach. For example one where we can vary the window size to determine more accurately either time or frequency. It is one of the advantages of the Wavelet Analysis: a windowing technique with variable sized regions. A Wavelet is a waveform of effectively limited duration that has an average value of zero.

3.1 Continuous wavelet transform

The continuous Wavelet transform is the scalar product of the signal with a scaled and positioned function :

$$X(a,b) = \frac{1}{\sqrt{a}} \int x(t) y \left(\frac{t-b}{a} \right) dt \qquad (16)$$

Where ψ is the mother wavelet, a the scale coefficient and b the position coefficient.
We loose the frequency reference because we don't work about the frequency of the analyzing function but on the size of the window. However we can still estimate the frequency by the value of the coefficient a. If a is low the wavelet is compressed and the frequency is high, if a is high the wavelet is stretched and the frequency is low.

. The bigger problem of the continuous Wavelet transform is the redundancy. Calculating Wavelet Coefficients at every possible scale is a fair amount of work and it generates an awful lot of data.

3.2 discrete wavelet transform

In order to eliminate the redundancy, we choose a exponential method in order to make the discrete transform :

$$\begin{cases} a = a_0^m \\ b = n.b_0.a_0^m \end{cases} \qquad (17)$$

Therefore, the discrete Wavelet transform is :

$$X(m.n) = \int x(t) y \; (a_0^{-m} - nb_0).dt \qquad (18)$$

In this paper we use the dyadic case where $a_0 = 2$ and $b_0 = 1$. If the transform is discrete, the mother Wavelet ψ stay continuous. There are a lot of Wavelet: Morlet, Haar, Daubechie, Mexican hat, etc. In our study we used only Haar and Daubechie (it is interesting to note that the Haar function is just the Daubechie function of order 1).

The interest of this type of transform, if the mother Wavelet is orthogonal, is the possibility to make the partial or total reconstruction of the signal and multiresolution analysis. This type of information is useful for the noise rejection and for a statistic work.

In order to do the decompositions we use Mallat algorithm [xx] with different Wavelets. The choice of the mother wavelet is important for the analysis. The figure 4 presents the level 1 of detail reconstructed on a noisy signal. The decomposition with an order 4 Daubechie Wavelet more readable than the decomposition with Haar Wavelet. The extraction of the signal versus noise will be easier with a Daubechie 4.

Another utility of the wavelet is the 2D analysis in order to make a statistical analysis. Presently we aren't working on this type of analysis.

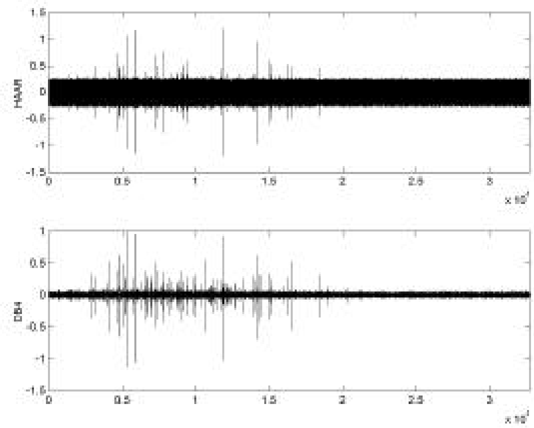

Figure 4: Comparison of decomposition between Haar and Daubechie 4 wavelets

4 Experimental case

We present a studie on a SKF bearing 6220 mounted on an induction motor of 3MW at 750 rpm. We measure the vibrations with a accelerometer Wilcoxon witch has a bandwidth from 1 Hz to 11 kHz. We are considering 3 cases:

1) bearing without defects,
2) defect on external ring,
3) defect on internal ring.

4.1 Time frequency analysis

The spectrogram on the signal of the vibration on case 1 (figure 4) does not give information. On case 2 (figure 5) and case 3 (figure 6) we can see vertical bands. The periodicity of these bands correspond at the characteristic frequencies of the bearing. These frequencies are given by equations 16 and 17:

$$f_{be} = \frac{Z}{2}\left(1 - \frac{D_r}{D_m}.\cos(\alpha)\right) f_{rot} \approx 51\,\text{Hz} \qquad (19)$$

$$f_{bi} = \frac{Z}{2}\left(1 + \frac{D_r}{D_m}.\cos(\alpha)\right) f_{rot} \approx 73\,\text{Hz} \qquad (20)$$

where: f_{be}: dynamic frequency of external ring,
f_{bi}: dynamic frequency of internal ring,
D_m: bearing mean diameter,
D_r: rolling element diameter,
Z: number of rolling elements,
α: contact angle,
f_{rot}: rotation frequency.

The aspect of the bands could give some information about the type of defect (in our case it is a hole diameter 1 mm deep 0.5 mm).

This type of representation of the single is not useful in the standard process but it is very interesting in the analysis after an alarm on an scalar indicator.

The time-frequency analysis contributes strongly to the qualification and the estimation of the defects. the periodicity and the aspect of the bands are characteristics of the defective element.

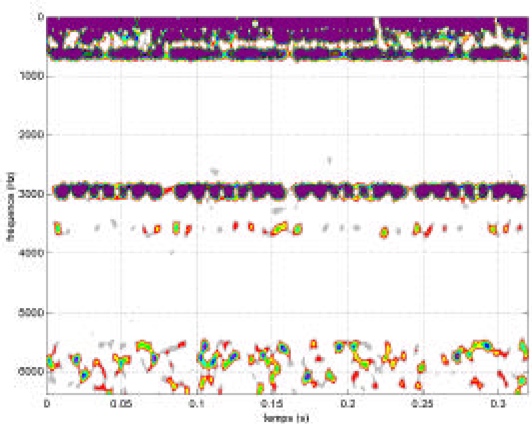

Figure 4: Spectrogram of the vibration on case (1)

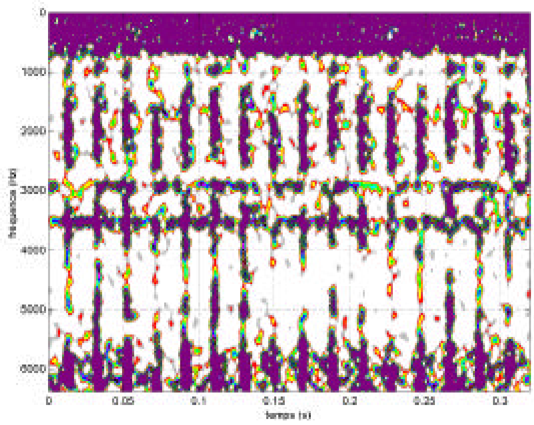

Figure 5: Spectrogram of the vibration on case (2)

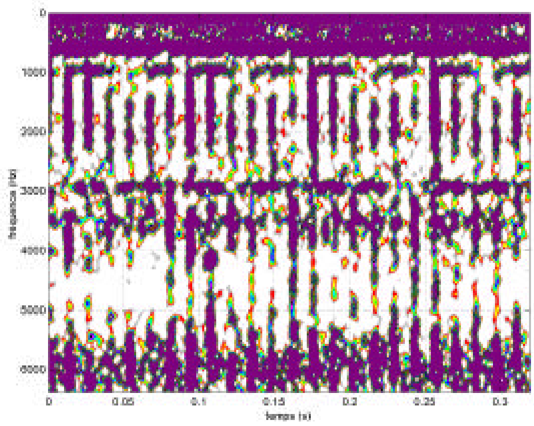

Figure 6: Spectrogram of the vibration on case (3)

4.2 Time-scale analysis

The wavelet analysis and multiresolution of the vibration permit to make a fast filtering. It can be interesting to use them in order to work on the resonance of the chocks.

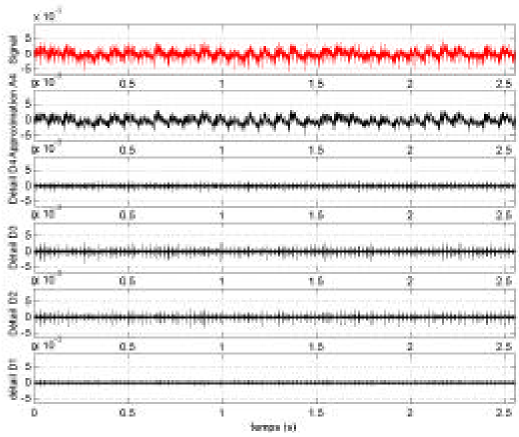

Figure 7: 4 level decomposition of the vibration in case 2..

Presently we use wavelets just to filter the signal. We don't use it as an expertise tools. The figure 7 presents the decomposition of the vibration signal in case 2. This decomposition is done with a wavelet DB4. the levels 3 and 4 are interesting to do apply some indicators like Kurtosis.

5 Conclusion

In this paper, we present the time-frequency and time scales transforms in order to analyse the vibrations on roller bearings.

In our process of expertise, the better tools are the times frequency transforms. They permit a qualitative and quantitative analysis of the defects. We use the wavelet only as a fast filter in order to apply statistic indicators on the filtered signal.

The work on the time frequency domain is an important part of our process of expertise. It is integrated on industrial process as a tool of predictive maintenance.

References

[1] Cohen L. "time frequency analysis", Prentice Hall, 1995.

[2] Mallat S. " A wavelet tour of the signal processing.

[3] Caironi C. "contribution a la maintenance predictive des machines tournantes par la mise en place d'outils de surveillance et d'évaluation des décharges partielles" Thesis IDPE, 25 oct. 2000.

[4] Brie D. "Modelling of the spalled Rolling Elements Bearing Vibration Signal: an overview and some new results", Mechanical and signal processing.